Journal of Culture and Values in Education

https://cultureandvalues.org

E-ISSN: 2590-342X

Volume: 7 Issue: 2 2024

pp.115-135

# Multilingual Students’ Solution Strategies in Solving Linear Programming Problems: A Case of National Curriculum Vocational Level 3 Mathematics Students

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**Article Info**
Received: February 8, 2024
Accepted: April 10, 2024
Published: July 30, 2024

10.46303/jcve.2024.15

**How to cite**
Machaba, F. M., Sipholi, K., & Motseki, P. (2024). Multilingual students’ solution strategies in solving linear programming problems: A case of national curriculum vocational level 3 mathematics students. *Journal of Culture and Values in Education*, 7(2), 115-135.
https://doi.org/10.46303/jcve.2024.15



**ABSTRACT**

This paper presents an exploratory case study that investigates multilingual students' solution strategies for linear programming problems at a technical vocational education and training (TVET) college in Gauteng. South Africa, like many developing nations, uses English as the language of instruction in technical vocational education and training in TVET colleges, despite the fact that many students lack the essential fluency to interact effectively with the curriculum. Where TVET lecturers and students speak the same home language, there is a disconnect between language policy and codeswitching, which is a frequent approach for achieving pedagogical goals. However, lecturers' training in multilingual realms in the classroom has frequently been framed in terms of linguistic issues, with a limited understanding of code-switching. The ability to combine two languages in the classroom in a systematic manner to promote learning has not been widely appreciated. Furthermore, codeswitching discourses are frequently veiled, with lecturers bringing the vernacular into the classroom. The authors of this study advocate for the discontinuity framework, which claims that in order to learn mathematics, pupils must first acquire the essential language. A purposive sample of 18 National Certificate (Vocational) Level 3 bilingual students was given the linear programming challenge. Data were gathered through a test and semi-structured interviews. The inductive content analysis, which was used for collection and analysis, revealed that students were unable to complete the linear programming task successfully because to a lack of relevant linear programming vocabulary (register) caused by language barriers. We thus recommend that lecturers use the mathematics register in their discussions with students about mathematics to demonstrate the complex and precise ways of expressing mathematical ideas; for example, lectures may re-voice their students' language representations so that these expressions more closely approximate the precision of the mathematics register.

**KEYWORDS**

Linear programming; national curriculum vocational; multilingual; discontinuity model.

## INTRODUCTION AND BACKGROUND

Language is widely accepted to play a vital part in each country's economic development. English dominates the local economy in South Africa, as it does in many developing nations, and as a result, the majority of South African technical vocational education and training (TVET) colleges utilize it as their language of teaching. The ability to communicate mathematically is essential for both teaching and learning (Setati, Molefe, & Langa 2008). The National Certificate (Vocational) (NC[V]) Subject Guidelines for Mathematics NQF Level 3, one of the main policy documents in South African higher education, recognizes the thinking individuals learn in mathematics as an ability to handle abstractions and an approach to problem solving (Department of Education [DoE], 2007). Language and discourse strategies in the classroom enable students to abstract mathematical concepts and relationships (Sfard, 2008). The use of language also creates a tension between students' interactions with the mathematical meanings of word problems and the mathematical processes required to solve them. As a result, language skill becomes an important aspect in comprehending the mathematical job. As a result, language proficiency is vital because mathematical abstractions are dependent on understanding the language in which they are presented (Sharma & Sharma, 2022).

Mathematics is taught and learned in South African technical vocational education and training (TVET) colleges using English as the language of teaching and learning (LoLT). Graven and Sibanda (2018) argue that the language of teaching, learning, and assessment is critical for acquiring mathematical comprehension and successfully interpreting examinations. As a result of the diversity of languages among students and lecturers, many TVET classes suffer from a lack of efficient communication (Turkan & de Jong, 2018). Lecturers and students have varying language backgrounds because they come from all sections of the country, and some come from outside South Africa.

In other colleges, many lecturers do not speak any of South Africa's indigenous languages (Brijlall & Kahiya, 2021). At the same time, students report that they believe they fail because they do not comprehend English (Lightfoot et al. 2022). They complain about not comprehending English as a language of instruction and learning. As a result, students' brains turn off when they see word problems. Linear programming (LP) is one area of mathematics in which students have difficulty learning word problems. Andriani and Ratu's (2018) research on LP demonstrates this difficulty. They discovered that students made errors in modelling using mathematical symbols, selecting a feasible zone, and failing to answer to the question posed. In support of this, Jaenal et al. (2023) discovered that students frequently struggle to solve linear programming questions due to a lack of understanding of the questions and trouble with arithmatic operations. As a result, students want the instructor to explain the topic in their native language and draw the equations for them (Brijlall & Kahiya, 2021).

When the instructor does not speak the indigenous language, there is almost no involvement. When lecturers and students share a shared home language, there is frequently a gap between lecturers and students share a shared home language, there is frequently a gap between language policy and practice, and lecturers and students use codeswitching to achieve pedagogical goals (Planas, 2018). The goal of this research was to investigate the role of multilingualism in NC(V) Level 3 students' solution techniques for handling LP issues in a multilingual classroom at a TVET college in Gauteng. To achieve this goal, we posed the question: to what extent is multilingualism feasible?

## LITERATURE REVIEW AND THEORETICAL FRAMEWORK

### *The relationship between mathematics and LOLT*

Most of the questions in linear programming are contextualised and are predominantly word problems that involve the translation of mathematical statements into symbolic form. The challenge of contextualised activities is that they tend to have long texts that students need to read, comprehend and interpret before they can be manipulated and represented in any other form. It is well known that learning mathematics is very similar to learning a language since mathematics has particular ways of speaking, reading and writing (Pimm, 2019).

Research has continuously argued that mathematics is a way of communicating (Pimm, 2019; Wilkinson, 2019; Zevenbergen, 2001). Wilkinson (2018, 2019) and Pimm (2019) use the notion of the mathematical register to indicate that mathematics has its own vocabulary that is made explicit through natural language. This argument suggests that language serves as a mechanism whereby mathematical concepts and ideas are communicated. Pimm (2019) argues that although mathematics always emerges in a natural language when spoken, it makes use of a different, rule-governed system which is independent of that of the natural language in which it can be read or spoken when written. Pimm (2019) refers to these dynamics as the dual nature of mathematics, i.e., the fact that mathematics is at once a medium and a message. For instance, at Grade 9 level, learners learn factorisation, and when they get to Grade 10, they use factorisation to solve equations. In Grade 9, learners learn factorisation as a message, but in Grade 10, it becomes a medium to solve equations. Zevenbergen (2001) argues that learners have to learn mathematics both as a language and as a discipline of knowledge for them to communicate mathematically within their communities. This finding is in line with the NC(V) Subject Guideline for mathematics (DoE, 2007, p. 2), which stipulates that students must be able to communicate using descriptions in words, graphs, symbols, tables and diagrams. This suggestion implies that learners face challenges of understanding the language of mathematics that is used to teach mathematics and the knowledge that they need to get from mathematics as a subject for them to make necessary representations.

### *The discontinuity models*

The discontinuity model defines mathematical learning as the acquisition of mathematics-specific language (Barwell, 2014; Moschkovich, 1996). This viewpoint emphasizes the acquisition of technical mathematical terminology or mathematical concepts as the primary means of learning mathematics. This prerequisite implies that pupils with a limited vocabulary of mathematical concepts are regarded as unskilled in mathematics. Learning a mathematical vocabulary is one of the practices that occurs in everyday mathematical discourses. Pimm (2019) claims that mathematics is a discipline with its own grammar. According to this reasoning, mathematics has its own language, register, and writing style that students must get familiar with.

The preceding perspective is consistent with certain previous comments on what constitutes mathematical learning (Barwell, 2014). For example, Barwell (2014) emphasizes that second language learners should concentrate on learning how to solve word problems, comprehend mathematical technical terms, and convert from English to mathematical symbols. In the LP job assigned to the students in this study, they were asked to begin by translating the mathematical statements into a symbolic form known as constraints or a system of inequalities. To construct such limitations, students must comprehend the ideas employed in LP, which are part of their mathematical language.

The discontinuity model, as defined by Moschkovich (1996, 2018), was then utilized as a lens to identify how pupils in the study formed a mathematical vocabulary. Knowledge of mathematical concepts (vocabulary) is one of the mathematical practices that pupils should be taught. However, this approach is insufficient for analyzing mathematical interactions (teaching and learning that includes debates and activities) in bilingual classrooms. This perspective is limited because it appears to ignore mathematical practices that students can bring with them to mathematical classrooms without utilizing mathematical jargon. This perspective ignores the informal mathematical knowledge that students and lecturers bring to mathematical classrooms.

When working with second-language students, more emphasis should be placed on the mathematical ideas they bring to mathematics classrooms rather than whether they use the correct vocabulary. It should be the lecturer's responsibility to ensure that students understand the correct mathematical notions while providing explanations that can be summarized in one mathematical concept. For example, students in my class have always struggled to supply an idea for an asymptote, although they can correctly explain what an asymptote is.

**Table 1.**

*Mathematics Instruction for Students Learning Mathematics Regardless of Lack of Proficiency in English*

| Emphasis | Descriptions |
|---|---|
| 1-Balancing conceptual understanding and procedural fluency | Teaching should balance student activities that address both significant conceptual and procedural information linked to a mathematical topic, as well as connect the two. |
| 2-Maintaining high cognitive demand: | Teaching should include high cognitive-demand math problems that maintain rigor across lessons and units. |
| 3-Developing beliefs | Teaching should help pupils build the belief that mathematics is sensible, worthwhile, and practicable. |
| 4-Engaging students in mathematical practices | Teaching should give students opportunity to engage in a variety of mathematical procedures. |

Source: Moschkovich (2019). Erath, Ingram, Moschkovich, and Prediger (2021), Schüler-Meyer

Meaney, Uribe and Prediger (2023).

Moschkovich (2018) contends that focusing on a student's failure to use a technical phrase may obscure how a student develops meaning for mathematical terminology or uses numerous resources, such as gestures, objects, or ordinary experiences. We may also overlook how the learner applies crucial features of competent mathematics communication that go beyond a vocabulary list. Making conjectures, engaging in constructive discussions, and providing explanations are all part of learning mathematics, in addition to learning terminology and technical terms.In her study, Moschkovich (2019) emphasized that instructional strategies should enhance these multilingual pupils' mathematical reasoning and sense-making abilities. She outlined four emphases, as seen in Table 1.

**Table 2.**

*Recommendations for Connecting Mathematical Content to Language*

| **Recommendations** | **Descriptions** |
|---|---|
| 1. Focus on students' mathematical reasoning rather than their precision in utilizing the LOI. | If the goal is to encourage student engagement in mathematical discussions and activities, discovering the source of an error is less important than listening to students and identifying the mathematical concepts in what they say. |
| 2. Shift your focus to mathematical discourse techniques and away from simplistic notions of language. | To summarize, instruction should shift away from perceiving precision as meaning utilizing the precise word and instead focus on how accuracy works in mathematical processes. |
| 3. Recognize the complexity of language in math classes and help students engage with it. | Teaching should acknowledge and strategically enhance students' ability to engage with this linguistic complexity. |
| 4. Consider ordinary language and experiences as gifts, not impediments. | Teaching must a) move away from monolithic notions of mathematical discourse and dichotomized views of discourse practices, and b) see daily and mathematical discourses as interdependent, dialectical, and related rather than mutually incompatible. |
| 5. Discover the maths in what students say and do. | Materials and professional development should support teachers so that they are better prepared to deal with language and mathematical content, in particular, a) how to uncover the mathematics in student contributions, b) when to move from informal to more formal ways of communicating mathematically. |

Source: Moschkovich, J. (2019). Erath, Ingram, Moschkovich, and Prediger (2021); Schüler-Meyer, Meaney, Uribe and Prediger (2023).

She stated that the primary goals for teachers teaching mathematics to students learning the Language of Instruction (LOI) are to teach for comprehension, to assist students in using multiple representations, and to assist students in communicating mathematical concepts using emerging and imperfect language. Because many resources indicate how to teach mathematics for comprehending and using multiple representations, Table 2's recommendations focus on how to relate mathematical material to language, specifically via "engaging students in mathematical practices" (Emphasis 4).

## RESEARCH DESIGN AND METHODOLOGY

This section describes the research design and methodology used to collect and analyze data. It also explores the ethical difficulties raised during the study. An exploratory case study research design was used. Swedberg (2020) defines an exploratory case study as a detailed investigation into empirical evidence collected over time from well-defined instances to describe a phenomena and the processes involved. We collected data from a cohort of NC(V) Level 3 multilingual students who had registered for a mathematics module at college and had completed and passed their Grade 12 mathematics examination, whereby the LOI is not their first, home or main language. Exploratory case study design was deemed appropriate because it enabled us to gain in-depth knowledge and explore their level of conceptual understanding of the LP concepts used in the tasks. The study was located within the interpretivist qualitative research paradigm, and face-to-face interviews sought to understand multilingual students' solution strategies when solving linear programming problems and the language practices used (Pham, Donovan, Dam & Contant, 2018). Students, therefore come from different language, social and cultural backgrounds. A purposive sample of 18 participants, made up of eight male students and ten female students, was selected because their achievement on the concept of LP was below 50%. By definition, purposeful sampling is an intentional selection of participants based on their abilities to provide the required information in relation to the research (Robinson, 2014). The average age of participants ranged between 17 to 19 years.

The study included students from a single site of a FET College in Gauteng. The campus is located in a Central Business District (CBD) in Gauteng and has the capacity to accommodate students from various provinces across the country. The college's students come from diverse linguistic, social, and cultural backgrounds. The campus has only eighteen (18) Level 3 pupils doing mathematics, with sixteen completing the task.

In this study, data was collected through written tasks and student interviews. The first author assigned a written task about linear programming. The lecturer assigned the work in front of the researcher. The first author graded and analyzed the written assignment. Two students were chosen specifically for reflection interviews based on their performance and level of conceptualization when responding to the exercise. These students scored the top and lowest on the written task. All interviews were audio recorded and transcribed.

In this study, descriptive validity was ensured by capturing all the students' responses to reflective interviews. These interviews were sought to establish what led them to respond to the task the way they did and to explore their level of conceptual understanding of the LP concepts used in the task. All utterances that were audio-taped during the reflective interviews were carefully listened to, and every word was transcribed without any meaning attached to it.

In this work, we employed contextual cognition theories and the discontinuity model to gain a general understanding of what it means to acquire mathematics and its terminology.

According to situated theory, participating in a community of practice improves mathematical learning, whereas the discontinuity model considers mathematical learning as the acquisition of mathematics-specific vocabulary.

According to Giardina and Newman (2014), there is a strong relationship between theoretical validity and construct validity. Construct validity is used to assess the credibility of any qualitative research. In this study, we ensured that data collection methods and research instruments that were utilised to collect and analyse the data serve as major sources for ensuring construct validity. All the transcripts were carefully analysed to obtain valid information on how students approach the LP tasks and how lecturers in multilingual classrooms support second-language students' engagement in meaningful mathematical participation.

***Written task***

Before being assigned to pupils, an LP problem from the national DoE's example question paper (DoE, 2008) was examined. The motivation for analyzing the task first was to determine the levels of cognitive demand that students are required to meet in order to properly complete the activity. Analyzing the assignment first helped to ease some of the potential concerns about the nature and/or type of tasks that would be assigned to the students during the study. We present the task below.

**Exemplar Question paper (June 2008)**

1 JJ Blocks, a block manufacturing company, is intending to **launch** a new and **improved** building block. For the manufacturing process, a mixture of two sand types A and B is required. At least 800kg, but not more than 1200kg of the two sand types are needed to prepare a batch of the new blocks. The mixture must contain at least 2kg of A to every 1kg of B, and at least 200kg of B must be used in the batch.

1.1 Present this information as a system of inequalities. (4)

1.2 Graph the system and indicate the feasible region. (5)

1.3 Use the graph to find the most economical mixture if sand type A costs R4/kg and sand type B costs R7/kg. Indicate the search line on the graph. (5)

1.4 Determine the minimum cost required. (2)

(DoE, 2008)

## RESULTS OF RESEARCH

Sixteen (16) NC (V) Level 3 students at Campus A took part in the written task. It took students about an hour to answer the task comprising four questions. An hour was a relatively long period since the task constituted only 16% of the entire question paper, especially considering that it was just a week after they were taught the section and were also told in time that we would come to give them a task on the section. The students worked individually to complete the task. The table below represents how students performed in the different questions of the task.

**Table 3.**

*Students' Performance in Different Questions of the Task*

| Question number | Solutions | Number of students |
|---|---|---|
| 1.1 | 4 inequalities correct | 0 |
| | 3 inequalities correct | 2 |
| | 2 inequalities correct | 4 |
| | Only 1 inequality correct | 6 |
| | 0 inequalities correct or no response at all | 4 |
| 1.2 | Correct feasible region shaded with 4 graphs correct | 0 |
| | Incorrect feasible region with 3 graphs correct | 1 |
| | Incorrect feasible region with 2 graphs correct | 2 |
| | Incorrect feasible region with only 1 graph correct | 5 |
| | Incorrect graphs or no graphs drawn | 8 |
| 1.3 | Correct cost equation, coordinates of the corner points of the feasible region, correct most economic mixture and search line | 0 |
| | Correct cost equation, coordinates of 4 corner points of the feasible region and most economic mixture | 0 |
| | Correct cost equation with an incorrect economic mixture | 3 |
| | Incorrect cost equation or no equation with an incorrect economic mixture | 13 |
| 1.4 | Correct minimum cost | 0 |
| | Incorrect minimum cost | 9 |
| | No attempt | 7 |

It is clear from the above table that no students stated all the correct systems of inequalities. As a result, no student drew all the graphs correctly, and consequently, no student represented the correct feasible region. Only two out of the sixteen students managed to get three systems of inequalities correct and indicate their feasible region as such. It is also clear from Table 3 that the majority of the students failed to develop systems of inequalities or constraints and, as a result failed to answer the rest of the questions that followed. In fact, some did not even attempt to answer them, especially Parts 1.3 and 1.4. Responses for Student 1 and Student 8 were also analysed, and these were the students who were interviewed. One student showed some level of conceptual understanding when responding to the task. This was Student 1 whom we have called Tshepo (pseudonym) in this study. Below is how Tshepo responded to the task:

*Tshepo's (student 1) responses (1.1 – 1.3)*

**Figure 1.**

*Tshepo's response to question 1.1.*

1 JJ Blocks, a block manufacturing company, is intending to launch a new and improved building block. For the manufacturing process a mixture of two sand types A and B is required. At least 800kg, but not more than 1200kg of the two sand types are needed to prepare a batch of the new blocks. The mixture must contain at least 2kg of A to every 1kg of B, and at least 200kg of B must be used in the batch.

1.1 Present this information as a system of inequalities. (4)

A + B ≥ 800kg

A + B ≤ 1200kg

B ≥ 200kg

A ≥ 2kg

**Figure 2.**

*Tshepo's Response to Question 1.2*

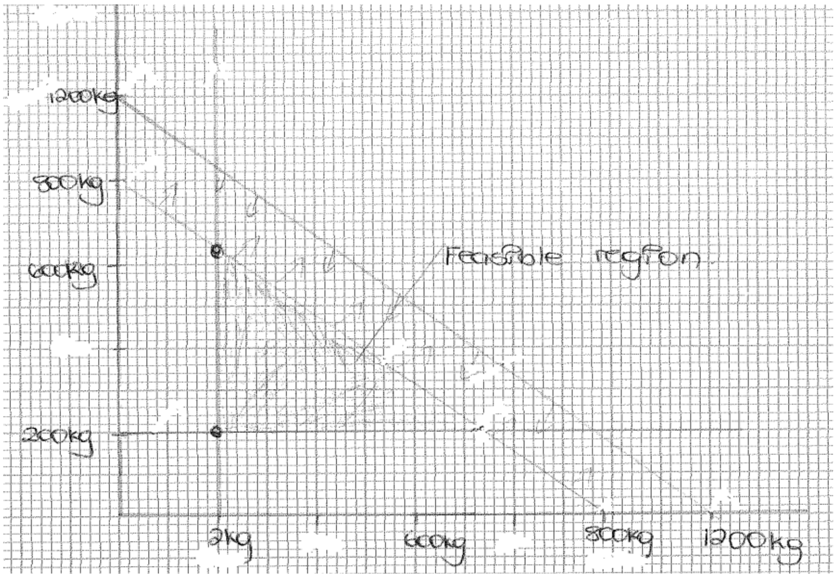
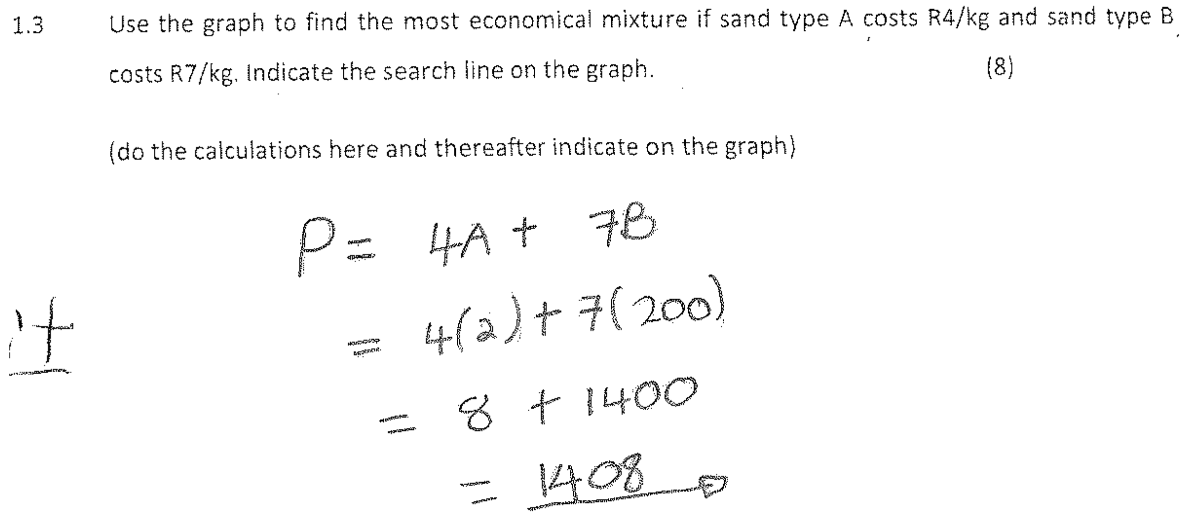


**Figure 3.**

*Tshepo's Response to Question 1.3*



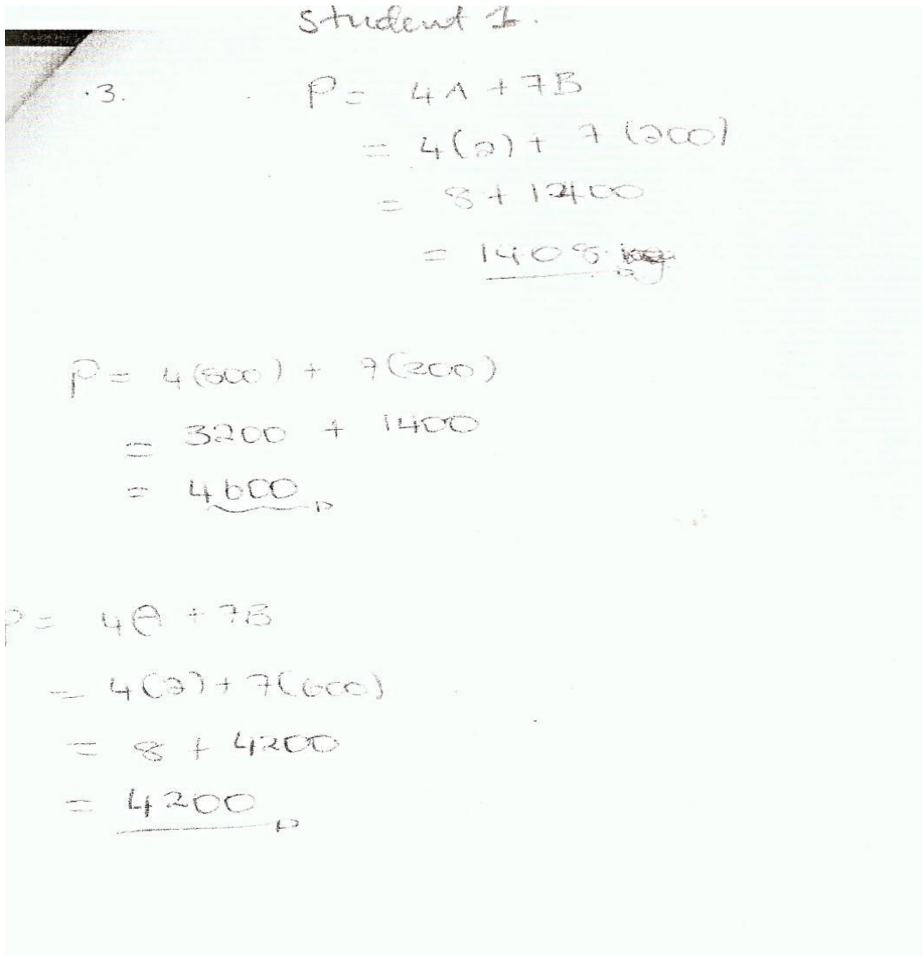
1.3 Use the graph to find the most economical mixture if sand type A costs R4/kg and sand type B costs R7/kg. Indicate the search line on the graph. (8)

(do the calculations here and thereafter indicate on the graph)

$P = 4A + 7B$
$= 4(2) + 7(200)$
$= 8 + 1400$
$= 1408$

**Figure 4.**

*Tshepo's Response to Question 1.3*



Student 1.

.3. $P = 4A + 7B$
$= 4(2) + 7(200)$
$= 8 + 1400$
$= 1408$ kg

$P = 4(800) + 7(200)$
$= 3200 + 1400$
$= 4600$

$P = 4A + 7B$
$= 4(2) + 7(600)$
$= 8 + 4200$
$= 4200$

It is clear that Tshepo obtained three constraints correctly (Question 1.1), and she drew correct graphs for the constraints (Question 1.2). All her graphs were shaded according to the constraints that she developed except for the graph of A + B ≥ 800kg, which was correctly drawn but incorrectly shaded. She did not name the axis and the interval for numbering of the horizontal axis, which was inconsistently done, i.e. 2kg, 600kg, 800kg and 1200kg. The student indicated the feasible region correctly according to her shading, but it was not positioned correctly due to incorrect shading of the graph mentioned above.

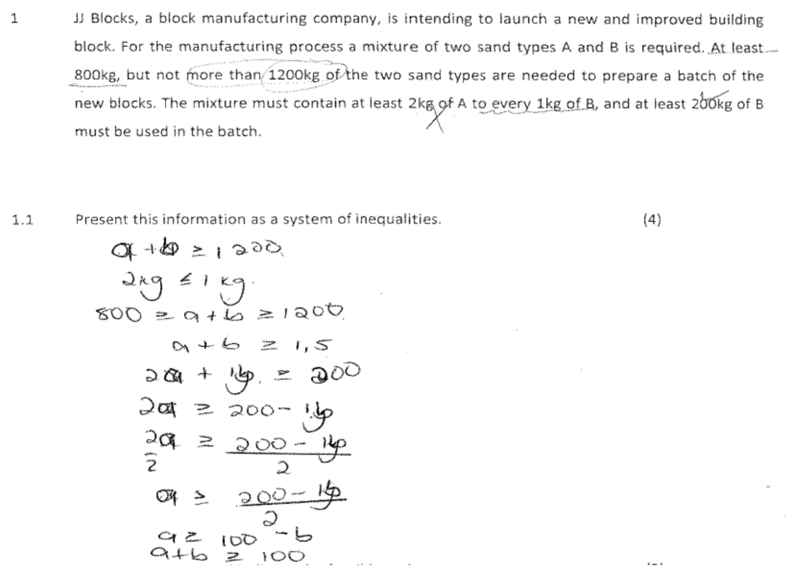
Question 1.3 required students to use the graphs to find the most economical mixture and to indicate the search line on the graph. The most economical mixture is represented by the coordinates of one of the four vertices or corner points of the feasible region. This question never required students to determine solutions algebraically but to indicate them on the graph. Tshepo used the algebraic method to answer this question, which suggests that she knows that to find the most economical mixture, she needs to determine the cost equation or objective function and the coordinates of the vertices of the feasible region first and then substitute the coordinates of vertices in the cost equation.

Tshepo also seemed to know that the solution to the most economical mixture would be the minimum cost of production, as she indicated in Question 1.4. What she did not seem to know was that the mixture had to be given as coordinates of a vertex. She followed the correct way of determining the minimum cost by substituting the coordinates of the vertex of the most economical mixture in the cost equation. What made her obtain the incorrect minimum cost was the inconsistency in numbering the axis, where she had as 2 kg instead of 200 kg. This mistake led to (2; 600) instead of (800; 200). When substituting her coordinates, she obtained R1 040 instead of R4 600.

Below are the solutions of Student 8, Taki (pseudonym), who was also interviewed. This student underlined some restrictions in the task statements but never obtained any correct answer for the whole task. Our reason for interviewing this student was to find her understanding of the task and the strategies that she used when responding to the questions in the task.

**Figure 5.**

*Taki's Response to Question 1.1*



1 JJ Blocks, a block manufacturing company, is intending to launch a new and improved building block. For the manufacturing process a mixture of two sand types A and B is required. At least 800kg, but not more than 1200kg of the two sand types are needed to prepare a batch of the new blocks. The mixture must contain at least 2kg of A to every 1kg of B, and at least 200kg of B must be used in the batch.

1.1 Present this information as a system of inequalities. (4)

$a + b \geq 1200$

$2kg \leq 1\ kg$

$800 \geq a + b \geq 1200$

$a + b \geq 1{,}5$

$2a + 1b \geq 200$

$2a \geq 200 - 1b$

$\frac{2a}{2} \geq \frac{200 - 1b}{2}$

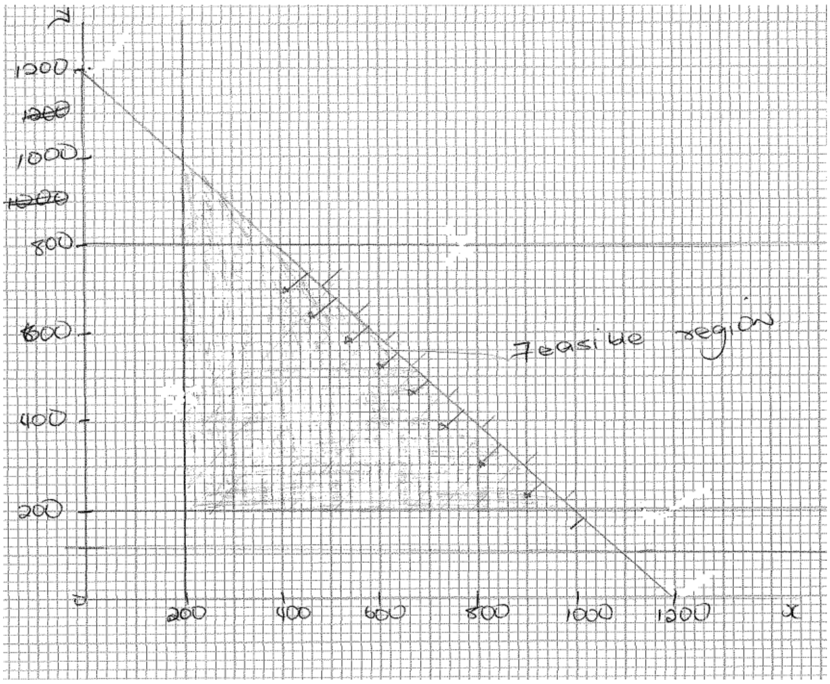
$a \geq \frac{200 - 1b}{2}$

$a \geq 100 - b$

$a + b \geq 100$

**Figure 6.**

*Taki's (Student 8's) Responses to Question 1.2*




Taki developed six constraints from the possible four. In the third constraint, there are two sets of operations or inequality signs. This response suggests that the student does not know that in an equation or system of inequality, there cannot be more than one set of mathematical operations. What followed after constraint number six was that she tried to solve for "a" or to express the constraint in its simplest form, which was also incorrectly done. From the six incorrect constraints that were developed, the student drew four graphs. Only one graph was drawn correctly according to the developed constraint, but it was shaded incorrectly. The other three graphs were not drawn according to the listed constraints, but they were shaded. The feasible region was indicated and labelled according to the graphs drawn. The labelling of the feasible region in the above graph suggests that the student understands the concept of a feasible region.

The above analysis of the student's responses to the task revealed that students find it difficult to develop constraints for LP tasks presented in the form of word problems. None of the students developed all constraints correctly. The analysis further shows that students are able to draw inequality graphs since they were able to draw the correct graphs for the constraints (correct and incorrect) that they developed. Most students could shade the inequality graphs accordingly and, as a result, could indicate the feasible region. The analysis also showed that students have problems when it comes to the naming of the axis and the use of consistent scaling. The next section presents reflective interviews with some of the above students who sought to find out how and why they responded the way they did.

***Analysis of reflective interviews***

For this article, two students from the 16 who participated in the written task were selected for reflective interviews. The selection for interviews was based on high and low achievement on the written task. Both students marked off some restrictions and concepts, which shows that they tried to read with understanding but performed differently in the task. As already indicated, the main objective for the interview was to get detailed insights into students' understanding of the questions as well as how they went about solving the task. For anonymity purposes, the selected students are referred to as Tshepo and Taki, which are pseudonyms. The interviews were semi-structured, with some pre-set questions prepared to guide the interview. Pre-set questions were developed from an analysis of how students responded to the written task. There were no pre-set categories for the analysis of the interview. The categories were developed as the analysis of interview data unfolded.

***Students' understanding of the problem***

The study involved multilingual NC(V) Level 3 students who are doing mathematics in a language that is not their first, home or main language. The written task given to students was an LP task presented in the form of a word problem. For such students to be able to solve word problems in LP successfully, they need to understand the problem. Understanding a problem needs students to read the problem with understanding, which involves fluency in the LoLT, students' inherent languages, as well as mathematical language. We look at how these language issues came out in the reflective interviews in the section below.

***The effect of the LoLT on the understanding of a written task***

Understanding is important when solving any mathematical problem presented in the form of a word problem. Students need to comprehend the mathematical word problem first before they actually access the mathematics embedded in the task. In the written task presented to students, questions were preceded by statements in a paragraph on which they were based. Should students fail to comprehend the statement, it follows that they will not be able to solve the task. During the reflective interviews that were conducted with students, they were requested to explain their own understanding of the task. Tshepo, who performed well in the task, gave the correct explanation of what the statement below meant, whereas Taki showed no understanding of the language used in the task statements. Below is Taki's explanation of how she understood the following statement:

> *Statement: JJ Blocks, a block manufacturing company, intends to introduce a new and improved building block. For the production process, a combination of sand types A and B is required. A batch of fresh blocks requires at least 800kg, but no more than 1200kg, of the two sand types. The mixture must have at least 2kg of A per 1kg of B, and at least 200kg of B must be used in the batch."*
>
> *Taki: They believe it's JJ Blocks, a manufacturing company that plans to launch a new building. In addition, a combination of sand types A and B is necessary for manufacturing processes. In addition, a combination of sand types A and B is necessary for*

*manufacturing processes. They state that sand types A and B are required for production, and that at least 800 kg but no more than 1200 kg of each sand type is required to make a batch of new blocks. It implies that it must not surpass. It means it must be 800 kg or less. And when they say not more than, it also means that it must not exceed. The mixture must contain at least 2 kg of A. They say that the thing that they mixed, it means that they mixed sand A and sand B, at least to be 2 kg. And then to every 1 kg of B means that they take 2 kg of sand and it also means that they take 5 kg of B, we will have 10 kg of A.*

Drawing from the above explanation, it is evident that Taki did not understand the task statement. For example, in her interpretation of the concept “t least” she said “*Ge bare at least bara gone eish de kg de se ka feta 800 kg”* (when they say at least, they mean that it must not exceed 800kg and this explanation is incorrect). The correct meaning of the concept “at least” is that the quantities must not be **less** than the quoted quantity. All instances where the student is trying to give her own explanation in Setwana are incorrect interpretations. The only correct statement is when the student was just reading and re-voicing the statement as it was presented in English. It was not surprising that in her responses to the task, Taki did not provide even a single correct answer. This response is mainly because she failed to comprehend the statements in the task, which would have assisted her to access the mathematics. It shows that if a student cannot understand the language through which the task is presented, they cannot access the mathematics embedded in the task due to the language barrier.

Language challenges that second language learners experience when dealing with mathematical problems have been a focus of research for a while now (Alt, Arizmendi & Beal, 2014). Alt and colleagues argued that there was a relationship between language and mathematics learning and teaching. In his seminal work, Pimm (2019) argues that the language of mathematics is an issue for all learners because mathematics has its own language. Mathematics and the language of learning and teaching are interdependent. The interdependency of mathematics and the language that is used to offer it has been aptly described by Zevenbergen (2001), who views mathematics as a language within a language because it cannot be communicated without a language to carry it. Alt et al. (2014) argue that language is a mechanism that learners use to communicate their mathematical ideas.

The argument presented above points to the fact that although mathematical communication is an issue for all students across the board, the problem is more complex for mathematics students who are still learning the language of learning and teaching (e.g. English).

The challenge that second language students experience when dealing with mathematical word problems is enormous because word problems require the competency of both mathematics and the language in which those problems are presented. This suggests that students need to understand the word problem first to be able to access mathematics. While it is true that having fluency in the language of learning and teaching mathematics does not necessarily mean that students will automatically find solving word problems easy, it is also true that fluency in the language of learning and teaching puts the students in a better position since they do not have to deal with lack of understanding of what the problem says. The above challenges manifested themselves when Taki could not explain what the statements in the task meant. Below is what Tshepo said about the task that she claimed to have enjoyed:

*Researcher: "Good! Eh! Which parts of these questions did you enjoy most?"*

*Tshepo: "I enjoyed the first question."*

*Researcher: "First question…First question. Oh! The one of developing systems of inequalities?"*

*Tshepo: "Yes!"*

*Researcher: "Why do you say you enjoyed that?"*

*Tshepo: "Oh! It…we used the same concepts in linear programming. It is the one that gives you the direction of what is going on. Maybe if you are…when you wanna draw the graph you need [to] know the system of inequalities because by not knowing them there is nothing that you can do after."*

*Researcher: Ok!*

*Tshepo: "And they are the concepts that the lecturer explained to us very well. We…I understand them very well. And then again they allowed us to go back in our groups, discussed it in our different languages, and so we understood very much better."*

*Researcher: Oh!*

*Tshepo: "After he called US, He, he explained them again in English."*

In the excerpt above, Tshepo stated that she appreciated the first question since she understood the concepts involved in the work. She stated that it was primarily because "We, I understand them very well." And then they let us go back into our groups and debate it in our various languages, which helped us understand much more." This quotation argues that using a student's native language improves conceptual understanding. Students' capacity to articulate their mathematical thoughts improved when they were allowed to use their native languages to supplement their learning and, most likely, expand their mathematical vocabulary. When learners modify vocabulary words in a variety of ways, they retain them more effectively (Moschkovich, 1996).

A variety of approaches can be employed, including the use of students' native tongues in group discussions. In the study, students were free to discuss concepts in their respective groups, explain them to the entire class in their languages, and then accomplish the problem. This technique has the potential to help pupils expand their mathematical vocabulary. Taki admits in this study that she struggles with word issues and so does not grasp them. When asked about the linguistic practices in class, she stated:

*Researcher: "But for you to draw those linear programming graphs you need to have those constraints, like you developed them here, of which in your case you said you ignore the inequalities signs and you put in an equal sign, and it becomes an equation right? How did you go about developing those constraints?"*

*Taki: "By translating those words into symbols."*

*Researcher: "Right, how difficult was it to translate those words into symbols? Bearing in mind that earlier on you said you have a problem with word problems."*
*Taki: "It is difficult, but at least our lecturer he always let us discuss in groups and when we do our group discussion we always use our language, but when we use and sometimes mmh when our lecturer use English, it becomes difficult, so ga re sa tlaloganyi re a mobitsa, [when we do not understand] we raise up our hands and then o a tla [he comes] and then he answers us and sometimes he uses our language to answer our question."*
*Researcher: "Does he know your language too?"*
*Taki: "Hmm."*
*Researcher: "You said in your group discussions you use your home language, right?"*
*Taki: "Yes."*
*Researcher: "Did your lecturer tell you to use your own language?"*
*Taki: "No, but when we use our language, he does not stop us. Sometimes, when we finish discussing, he will say explain to the whole class in your language what you understand. Like this word in linear programming or a classwork or one step."*
*Reseacher: "Ok, as he is explaining concepts in front is, he using English only, or he uses other languages as well."*
*Taki: "He only uses English. But he can only say some few words in Setswana in the whole period and nna sometimes a ke mo understandi[and sometimes I don't understand him."*

Taki also affirms the fact that the lecturer allows them to use their home languages. She did not obtain any answer correctly in the written task, and she admits that these concepts are difficult for her. What is interesting is her claim is that when they discuss her group and their home languages, she understands. She understands the concepts and can largely do tasks during group discussions since they discuss occur in their home languages. She also has a challenge with understanding the lecturer when he teaches in English. This challenge was evident when she said "He only uses English. But he can only say some words in Setswana in the whole period and nna sometimes a ke mo understandi" [and sometimes I don't understand him]. This statement shows that Taki does not always understand the lecturer when he explains in English, but when he code-switches to Setswana, she understands. It shows that she lacks proficiency in the LoLT. Her challenge is consistent with Danesi (as stated in Botes & Mji, 2010), who contends that students who are taught in a language other than their mother tongue are unlikely to achieve academic success, not because they are less capable, but because of an artificial linguistic difficulty.

## DISCUSSIONS

In NC(V) Level 3, test questions are primarily presented in the form of word problems using certain linear programming ideas. The analysis in this article demonstrates that students' approaches to linear programming assignments are impacted by how they were taught the topic. This finding is consistent with Schleppegrell (2007) and Snow and Uccelli (2009), who state

that students' familiarity with the classroom language, teachers' preparation for teaching mathematics through language, and teachers' preparation, which includes mathematics language learning, limit students' opportunities to learn linear programming in the classroom.
To perform well in linear programming, students must first comprehend these notions, which are part of the mathematical lexicon. The study's findings indicate that providing students with language opportunities in the classroom should not be limited to teaching vocabulary; rather, more emphasis should be placed on strengthening rich discourse practices such as explaining meaning, constructing arguments, and justifying procedures (Moschkovich, 2015). This notion is consistent with the characteristics of instructional approaches that promote conceptual understanding (Sharma & Sharma, 2022). As a result, language is critical for expanding children' options to learn mathematics. A learning environment and socio-mathematical norms that favorably support the active participation of all students in rich classroom discourse practices, such as explaining and disputing (Ingram, Andrews & Pitt, 2019).

The task consisted of sentences that conveyed the problem's requirements or limits, as well as some basic LP ideas. Students were supposed to apply the LP principles in the restrictions to create algebraic constraints or a set of inequalities. The findings also demonstrated that students did not understand the mathematical ideas employed in the challenge and hence did not apply them while solving it further. Learning mathematical language or concepts is one of the practices that should occur in everyday mathematical discourses.

Students' failure to understand underlying mathematical concepts embedded in linear programming tasks contradicts Erath et al.'s (2021) design principles for teaching and learning contexts, which include (1) engaging students in rich discourse practices, (2) connecting language varieties and multimodal representations, (3) including students' multilingual resources, and (4) comparing multilingual language pieces to raise students' language awareness. The design philosophy of engaging students in rich discourse activities includes communicative practices like codeswitching and translanguaging. However, new research (Schüler-Meyer et al., 2019) provides examples of using various languages as a source of meaning-making in a mathematics classroom. At the teaching practice level, Ingram et al. (2019) propose the introduction of linear programming assessment problems that support scaffolding using language.

In fact, Pimm (2019) claims that mathematics has its own grammar. This suggestion implies that mathematics has its own language, register, and writing style that students must get familiar with.

## CONCLUSIONS

In this study, students had to translate the given restrictions into symbolic form. This form of representation is called constraints. For students to succeed in responding to the given task, they needed to develop the constraints first, then draw some graphs based on the developed constraints and represent the feasible region, and, thereafter, answer other questions based on the graphs. This means that if students lack the vocabulary of the concepts used in the restrictions, they may not develop the correct constraints and, as a result, may not be able to solve the task as a whole. Barwell (2014) emphasizes that second language learners should prioritize learning how to solve word problems, understand mathematical technical terms, and convert from English to mathematical symbols.

During the reflective interview, Tshepo showed a clear understanding of the linear programming concepts and the restrictions contained in the statements of the task; hence, it was not surprising that she performed well in the task. On the other hand, Taki failed to share with the researcher what she understood by the statements in the task. She could not even explain in her own home language. All the explanations she gave were incorrect. It was also not surprising that she performed so poorly in the task, and we attribute her poor performance to a lack of conceptual understanding. Lecturers need to use different teaching strategies that will enhance the acquisition of an adequate mathematical vocabulary among students.

While this perspective presents one mathematical practice that should be instilled in learners, this tool alone was never an appropriate tool to analyse mathematical interactions and participation that happened during the teaching of linear programming in this multilingual classroom. Learning of mathematics goes beyond the acquisition of vocabulary or technical words to include elements such as making conjectures, engaging in constructive arguments and giving explanations.

We therefore recommend that lecturers modify and adjust their instructional tactics so that students engage in mathematical debates and use their whole communicative repertoire, including everyday language and the mathematics register (Wilkinson, 2018). Lecturers must prepare to help students grasp and analyze mathematical literature. Lecturers should design instructional environments in which students are engaged in demanding mathematical tasks and mathematical discourse, as well as able to interpret texts, assessments, and directions utilizing their entire communication repertoire, which includes writing. According to research, when students are asked to collaborate on tough, open-ended activities, they create and convey mathematical knowledge and reasoning.

Tasks should provide students with various opportunities to develop mathematical concepts and demand more than just employing numbers and computations, repeating procedures, or manipulating symbols. Lecturers and professors may choose to establish situations in which students demonstrate their conceptual understandings by explaining and justifying their solutions to difficulties in their own words first. The key criterion is for students to develop mathematical concepts for problem solving, generate different representations of those ideas, then transfer that process onto language expressions, eventually complying to the mathematics register's standard expressions.

Lecturers have the opportunity to help students through the problem-solving process and encourage them to be creative in their quest for understanding. According to Wilkinson (2018), in order to fully engage students in mathematical problem-solving, teachers may benefit from creating situations in which students use their broad repertoires of mathematical knowledge and skills to instructional objectives. Lecturers should use the mathematics register in their discussions with students about mathematics to demonstrate the complex and precise ways of expressing mathematical ideas. For example, teachers may re-voice their students' language representations to more closely approximate the precision of the mathematics register (Wilkinson, 2018).

This article highlighted how students' approach for solving linear programming are impacted by how they were exposed to the issue in the classroom. Lecturers in linear programming must adapt and change their instructional tactics so that students are better prepared to engage in mathematical debates that use full communicative repertoires, ordinary language, and the mathematics register associated with linear programming discourse.

This study was limited to eighteen NC(V) level 3 mathematics students participating in a mathematics course, namely linear programming, at a single TVET College in Gauteng. Extensive study can be undertaken on multilingual students' problem-solving skills in various mathematics topics, NC(V) levels, TVET colleges, Gauteng Province, and other provinces.

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