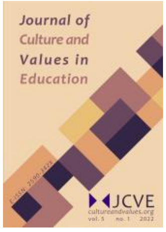

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Cultural Transitions in Mathematical Discourse: Unveiling Mathematical Writing Hurdles during the Rite of Passage

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ABSTRACT

This qualitative case study delves into the intricate landscape of mathematical writing challenges faced by first-year university students undergoing the critical transition from school-level to university-level mathematical discourse. Conducted at a prominent South African university, the research, employing a purposive sampling technique, engaged thirty first-year mathematics students. Guided by the community of practice theory by Lave and Wenger, alongside van Gennep's rite of passage analytical lens, the study sought answers to the question: What are the mathematical writing challenges encountered by first-year university students during their rite of passage period? Thematic analysis, informed by Adu's (2019) coding framework, was utilized to systematically examine common themes and patterns within the qualitative data. The findings illuminated key hurdles during this transitional phase, prominently including the inconsistency in working with mathematical notations, erroneous use of universal and existential quantifiers, and a notable confusion between the acts of illustrating and proving in mathematical contexts. In response to these challenges, the study advocates for the explicit incorporation of mathematical writing instruction to scaffold students during this rite of passage. Furthermore, it recommends a shift in the emphasis of first-year mathematics courses—suggesting a redirection from a content-centric approach to one that prioritizes the cultivation of students' new identities. This entails focused attention on teaching the customs, traditions, and adept ways of constructing and articulating mathematical proofs in the university context. The implications of this study extend beyond the immediate challenges identified, offering actionable recommendations to enhance the pedagogical strategies employed in the crucial transition period for first-year mathematics students.

KEYWORDS

Cultural Transition; discourse; mathematical writing; community of practice; rite of passage.

INTRODUCTION AND BACKGROUND

In the intricate landscape of mathematical education, the significance of effective communication skills cannot be overstated. As posited by Morgan et al. (2014), the development of these skills is imperative for comprehending and mastering mathematical concepts. Mathematics, akin to any language, possesses its own grammar, vocabulary, idioms, and syntax, shaping a unique discourse that extends beyond numerical values and equations (Setati, 2005). Recognizing the pivotal role of writing in mathematics, the National Council of the Teachers of Mathematics (NCTM, 2008) advocates for the integration of writing as an integral part of mathematical learning, challenging the conventional notion of it being a mere supplement. Transitioning from the familiar terrain of school mathematics to the uncharted territories of university-level discourse proves to be a formidable rite of passage for students. The shift brings forth challenges that cast shadows on the academic journey, causing consternation among both students and educators. Even those students who were once lauded for their prowess in school-level mathematics find themselves grappling with the complexities of first-year university mathematics (Chandrasegaran, 2013; Di Martino & Gregorio, 2019). Meehan et al. (2018) delve into the psychological dimension of this transition, exploring how high-achieving students' sense of belonging to the mathematical community is profoundly affected during the move from secondary to first-year of university mathematics.

Amidst these challenges, the importance of explicit teaching of mathematical writing becomes apparent. As emphasized by Matabane et al. (2022), university mathematics lecturers play a pivotal role in mitigating difficulties encountered by students by purposefully intertwining symbols, images, and nominalizations within the fabric of their teaching methodologies. The crux lies in not merely deciphering mathematical language but in articulating it effectively—a skill set that proves transformative during the rite of passage from school to university mathematics. Drawing from the insights of the NCTM, it becomes evident that writing is not just a peripheral aspect but a gateway to deeper comprehension and mastery of mathematical concepts (NCTM, 2008). This sentiment resonates with the stance of the Department of Basic Education (DBE) in South Africa, where learners are urged to cultivate a precise command of mathematical language and express their ideas adeptly through words, graphs, and symbols (DBE, 2011). However, as students make the leap to university, the dynamics of mathematical writing undergo a paradigm shift, demanding the construction of formal definitions and the adept use of formal language and reasoning (Clark & Lovric, 2009).

This study embarks on an exploration of the labyrinthine challenges encountered by first-year university students during the rite of passage, focusing on the transformative period marked by the evolution of mathematical writing. The central question guiding this inquiry is: What are the inherent challenges in mathematical writing faced by first-year university students during the rite of passage? In dissecting the intricacies of this transitional phase, we seek to unravel the layers of difficulty that conceal themselves within the folds of mathematical discourse, shedding light on the hurdles that impede a seamless transition for students into the realm of university-level mathematics.

LITERATURE REVIEW

Writing and Learning Mathematics

To increase students' knowledge and provide teachers with an understanding of their students' thinking is one of the most frequently mentioned pedagogical justifications for employing writing in mathematics classes (Morgan, 2001). Writing in mathematics serves as a tool for negotiating mathematical context and text in order to build mathematical knowledge and as a way to communicate and deepen mathematical understanding (Wallace et al., 2004). Teachers can diagnose misconceptions in their students' mathematical thinking, acquire insights into their mathematical reasoning, and evaluate their own teaching by having students write in mathematics classes (Martinez & Dominguez, 2018). When writing is integrated into mathematics instructions, students deepen their mathematical knowledge and learn mathematics concepts with deeper understanding (Boero et al., 2008; Durand-Guerrier et al., 2012; Kuzle, 2013). When students were given opportunities to write in mathematics classrooms and integrated mathematical language and natural language, weaving between images, words and symbols, they were motivated to learn and their mathematical communication improved (Matabane & Seo, 2021). For instance, Boero et al. wrote about the use of natural and symbolic languages in mathematics and claimed that "students can only perform in a satisfactory manner if students become comfortable enough with normal language in the suggested mathematics exercises" (2008, p. 262).

Transition from School to University Mathematical Writing

The transition from secondary school to university mathematics has caught the attention of many researchers (Bampili et al., 2017; De Guzman et al., 1998; Di Martino & Gregorio, 2018; Gueudet, 2008, Matabane et al., 2022; Tall , 1981). Seminal work was conducted by Tall (1981), investigating cognitive discontinuities in school-university mathematical learning transition. Inspired by Tall's work, De Guzmán et al. (1998) identifed three categories of students' challenges during the transition period: epistemological-cognitive, sociological-cultural, and didactical. Gueudet (2008) categorised organisation of knowledge, proofs, didactical disposition and mathematical communication as transitional challenges. Di Martino and Gregorio (2018) established and examined the so-called "first time phenomenon", which is the psychological response of successful individuals to their first instance of mathematical failure. By utilising the communities of practice (Wenger, 1998) theoretical framework, Bampili et al. (2017) examined how social and institutional factors influence the emergence of a new identity for first-year mathematics students. To help students to transition better, writing should be a central part of teaching and learning mathematics and students should be taught different ways of communicating mathematics, including the use of scripted words (Matabane et al., 2022, Seo, 2015).

Theoretical Lenses

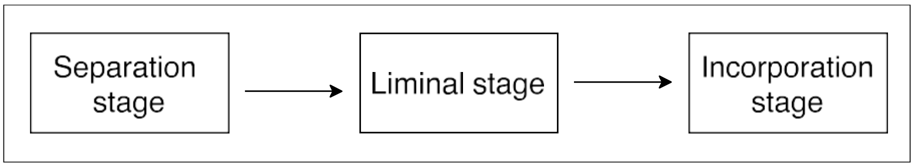
In his book, *Les rites de passage*, Belgian anthropologist and ethnographer Arnold van Gennep claimed that the process of moving from one group to another, which he referred to as "riding the passage", characterised certain stages in an individual's life (van Gennep, 1960, p. 12). Since the publication of his major work, the concept of rites of passage has become well-established in ethnography and anthropology and used to analyse transitional rituals, including childbirth, loss, and marriage. The goal of rites of passage, in the words of van Gennep, is "to assure a change in circumstances or transition from one magico-religious or secular group to another" (1960, p. 11). He argued that transitional rituals involved must be understood in their entirety and that all share a tripartite structure comprising the beginning (rites of separation), middles (rites of liminality) and the ends referred to as rites of incorporation (see Figure 1). This is not a process without challenges and risks. As Mary Douglas observed, van Gennep "saw society as a house with rooms and corridors in which passage from one room to another is dangerous" (1966, p. 6). Thus, the transition between the stages is marked by a cognitive and affective crisis. However this crisis is required for the transition to occur; otherwise, the person will be unable to integrate into the new group.

Like Arnold van Gennep's construct of riding, the passage resonates with Lave and Wenger's (1991) theory of community of practice. For instance, legitimate peripheral participation (LPP) outlines how novices develop into seasoned participants and then senior members of a community of practice or collaborative project over time (Lave & Wenger 1991). The idea that learning is primarily social and closely tied to a person's developing identity within a community of practice is at the core of Lave and Wenger's (1991) theory of community of practice. Di Martino et al. (2022) described the transition from school to university community as a rite of passage.

In this study, students have been separated from the school community to join a new community at the university. The separation stage (see Figure 1) is understood as the stage where students are moving away from school ways of writing mathematics to university ways of writing. The students have left (separated from) the school where they have been successful at writing mathematics to join the new context. During the liminal stage, the students are neither here nor there. The students do not belong either to the school mathematics context or to the university context. The liminal stage includes the last part of high school to the first part of being at university, and is characterized by unavoidable crisis (Clark & Lovric, 2008). At this stage, known mathematical routines are challenged (Clark & Lovric, 2008) and first-year students are subjected to a journey that divorces them from the world they knew, entering the liminal domain and ritualised to a find a place in the new community. The students are transformed through rituals, before arriving in the new world and university ways of writing mathematics and constructing logical sound and coherent mathematical arguments. It is at the liminal stage that the students start negotiating what it means to write mathematics within the university community. The liminarity passage includes a cognitive shift from informal to formal language and reasoning in mathematics (Clark & Lovric, 2009; Di Martino & Gregorio, 2019). Finally, the incorporation stage is when the students now understand ways of writing mathematics at university and become immersed into the university mathematics community. In the incorporation stage, students become fully fledged members of that university mathematics community. In this study, we looked at the challenges faced by students during this rite of passage.

Figure 1.

The Three Stages of the Rite of Passage Identified by van Gennep (1960)



METHODOLOGY

This qualitative case study was designed within the interpretivist paradigm, employing a naturalistic methodology to ensure a comprehensive exploration of the research questions. To enhance the representatives of the sample, purposive sampling was employed, allowing for the deliberate selection of thirty first-year mathematics students who could offer a nuanced perspective on the issues under investigation (Leedy & Ormrod, 2005). This group consisted of 18 females and 12 males ranging from ages 17 years old to 38 years old. Of this group, 26 participants were aged between 17 years old to 20 years old, and 4 were between 21 years old to 38 years old. Of the participants, 16 were specializing in the senior and further education and training (FET) phase, while 14 were in the intermediate phase (IP). The inclusion criteria were stringent, encompassing only those students who were not only in their first year at the designated university but were also first-time registrants with no prior enrollment at any other university. Additionally, the study exclusively focused on first-generation students (first ones in their families to go to university), ensuring a distinct lens through which to examine their experiences as trailblazers in their families attending university.

Data collection involved a meticulous blend of methods, combining the analysis of students' assignment scripts with semi-structured interviews. In the initial week, during the separation stage, students were tasked with demonstrating their understanding of the properties of natural numbers, specifically proving the well-known result that the sum of two odd numbers is always an even number. Subsequently, after four weeks of acclimating to university norms (liminal stage), a second task was administered, maintaining continuity by addressing the section on numbers. This task required students to define odd and even numbers and prove that the product of two odd integers is always an odd integer.

To ensure the reliability of the interview questionnaires, a rigorous review process was undertaken, involving mathematics teachers and researchers who scrutinized the questions for clarity, standardization, and specificity. The actual scripts of the students were analyzed to validate reliability and trustworthiness. Interviews were meticulously conducted, recorded, and transcribed verbatim, contributing to the creation of a robust and valid dataset. Ethical considerations were paramount, with a commitment to ensuring anonymity and confidentiality for participants. Pseudonyms were exclusively used in reporting the study results.

Thematic analysis, guided by Adu's (2019) coding framework, was employed to systematically examine common themes and patterns in the qualitative data. To align with the conceptual framework of van Gennep's (1966) three stages of the rite of passage, data analysis unfolded in three distinct stages. The first stage involved an examination of students' scripts during the initial week of university entry, focusing on understanding school-specific writing conventions (separation stage). The second stage delved into the students' writing a month after their arrival at the university (liminal stage). Finally, the third stage analyzed students' scripts three months into the course (incorporation stage). Subsequent to each stage, interviews were conducted with the students to elucidate the reasoning behind their written responses, echoing the insights of Hyland (1968) and Plato (1892) about the limitations of written texts in capturing the intricacies of a writer's thoughts. These interviews were deemed essential to bridge the potential gap between written expression and the underlying cognitive processes of the students, thereby ensuring a more comprehensive understanding of the data

RESULTS AND DISCUSSION

In analyzing students' responses, three themes emerged: confusing illustration to mean proof, inability to define numerical context, and inconsistent use of mathematical notations. In the first week of being at university (separation stage), the students were given task 1 where they needed to prove that the sum of odd integers is always an even integer (see Appendix A). In task 1, the main challenge for students was confusing illustration to mean proof.

Confusing Illustrations as Proofs

Lerato's response showed confusion between the rules of proving and those of illustrating. He used specific numbers to prove the general case. Although Lerato presented a clear table to show that every time he adds two of the odd numbers, in his table the answer is odd.This cannot be classified as a proof (Di Martino & Gregorio, 2019). The challenge is that it is not possible for Lerato to include all possible odd numbers in his table, and therefore he cannot claim his results are always true. Even if Lerato were to give a table consisting of the first 1000 natural numbers, that will still not constitute a proof as the results may fail to be true from the number 1001 onwards. In the university mathematics discourse, the use of counter examples can only be used to disprove the results and not to confirm them (De Guzmán et al., 1998).

Simar to Lerato, Dimpho used a series of numbers to claim that two odd integers added together will always equal an even number. Her explanations showed that she is not cognisant of the fact that numbers cannot be used to prove general results (Di Martino & Gregorio, 2019).

Figure 2.

Lerato's Response to Task 1

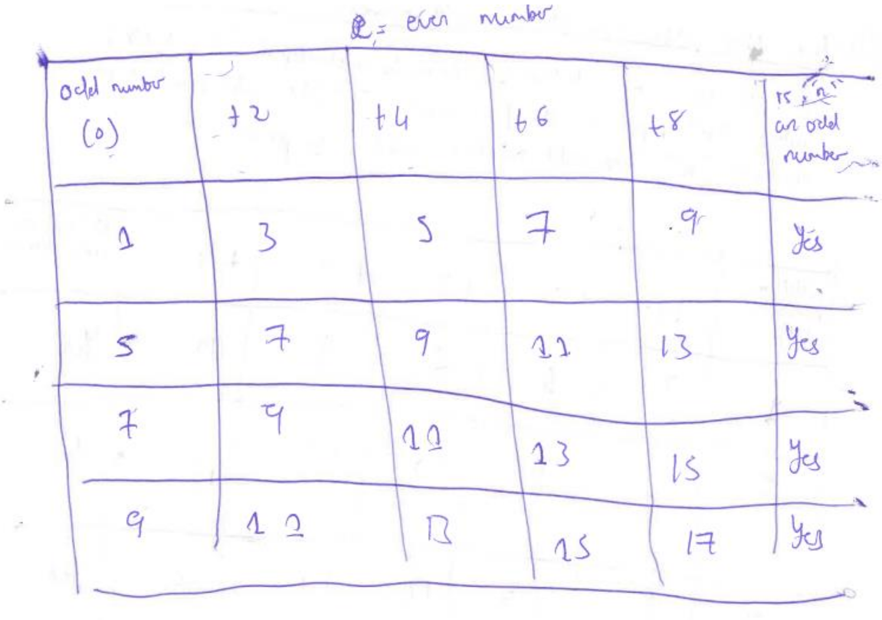
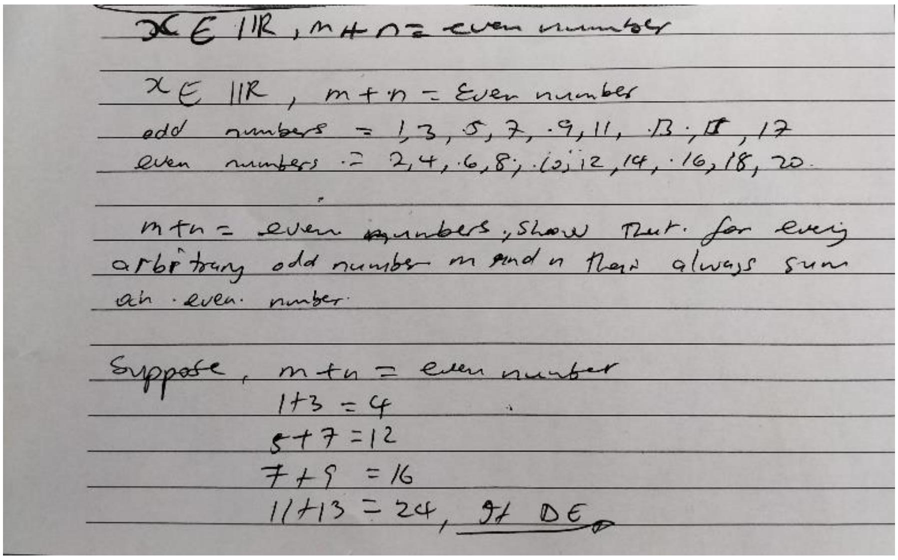


Figure 3.

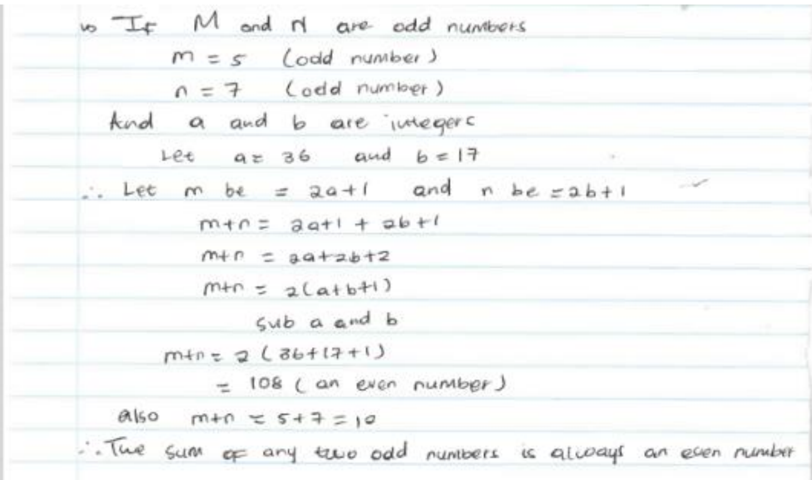
Dimpho's Response to Task 1



Percy started his proof by using specific numbers M and N. Although the question was clear that two given numbers are odd, Percy started his proof with a conditional statement: "If M and N are odd numbers". The use of "If" was not necessary in this case because the numbers are given to be odd. The correct vocabulary could be since or because M and N are odd. Again he claimed that if M is an odd number, then m=5. Similarly, he claimed that if N is an odd number, then n=7. Percy's solution illustrated more writing challenges. Firstly, when he started his proof he used upper case M and N. However, in the middle of his proof he used lower case m and n to represent the same odd numbers. Secondly, at the beginning of his proof the numbers M and N were introduced as randomly picked odd numbers, but in the next line they were specified as m=5 or n=7. Percy's proof was more clumsy as it used both specific numbers to prove as well as some attempt to define odd numbers as being of form 2a+1 and 2b+1. He attempted to bring the general definition of odd numbers but messed up the whole proof by specifying a to be 36 and b to be 17.

Figure 4.

Percy's Response to Task 1



If M and N are odd numbers

$m = 5$ (odd number)

$n = 7$ (odd number)

And a and b are integers

Let $a = 36$ and $b = 17$

$\therefore$ Let m be $= 2a+1$ and n be $= 2b+1$

$m+n = 2a+1 + 2b+1$

$m+n = 2a+2b+2$

$m+n = 2(a+b+1)$

sub a and b

$m+n = 2(36+17+1)$

$= 108$ (an even number)

also $m+n = 5+7 = 10$

$\therefore$ The sum of any two odd numbers is always an even number

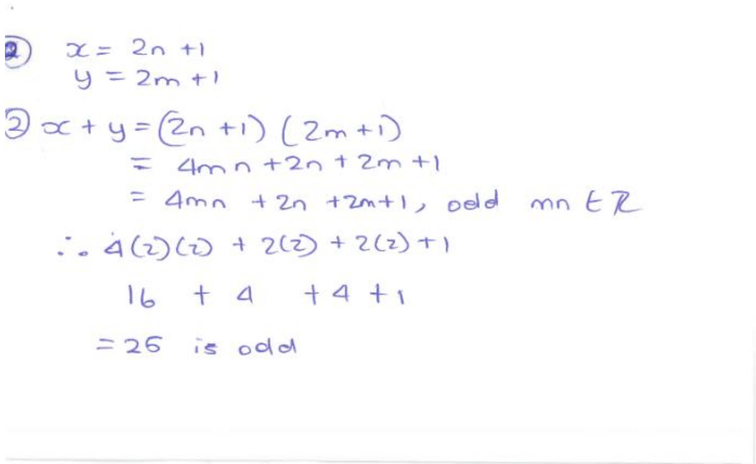
Inability to use Definition and Specify Numerical Context when Constructing Proof

A month into the course (liminal stage), and having discussed students' responses to task 1 two weeks earlier, the students were now given the second task, still within the domain of numbers. Students were asked to prove that the product of two odd numbers is always an odd number. For task 2, students were now able to distinguish between the rules of proving and those of illustrating. At this point, they were able to define the concepts before using them. Their main challenge was to use the given definition in the problem-solving process and to define or make explicit the numeral context that is assumed when presenting their definitions and constructing their proofs.

In starting his proof, Percy defined the odd numbers $x = 2n + 1$ and $y = 2m + 1$. This was a good start as the student is aware that he needs the general definition of an odd numbers to convincingly prove that the product of any two randomly picked odd numbers is always odd. However, the numerical context of n and m was not specified. Because of Percy's failure to clearly specify the numerical context of n and m, there is no guarantee that $x = 2n + 1$ and $y = 2m + 1$ are odd numbers. For example, if $n = 0.5$ and $m = 0.5$, both x and y will be even numbers. Then the whole process lost meaning due to non-specification of the numerical context. Again, on the left-hand side of his solution, Percy wrote "$x + y$", but on the right-hand side x and y were multiplied. Moreover, as Percy concluded that the product of two odd numbers is odd, he did not make his conclusion by showing that the product satisfied the definition of an odd number; instead he moved from generalising and choosing $m = 2$ and $n = 2$, spoiling the whole proof.

Figure 5

Percy's Response to Task 2



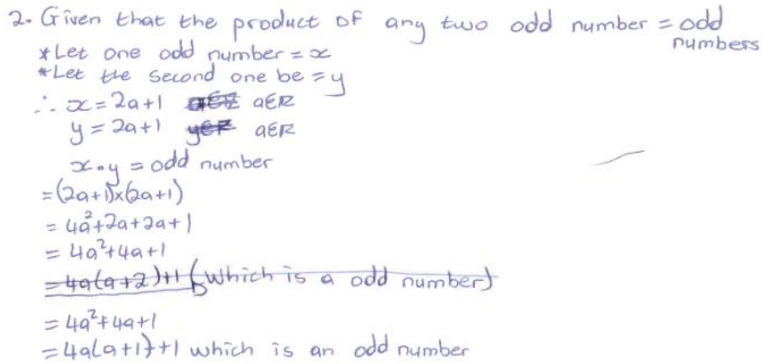
2) $x = 2n + 1$
$y = 2m + 1$

2) $x + y = (2n + 1)(2m + 1)$
$= 4mn + 2n + 2m + 1$
$= 4mn + 2n + 2m + 1$, odd $mn \in R$
$\therefore 4(2)(2) + 2(2) + 2(2) + 1$
$16 + 4 + 4 + 1$
$= 25$ is odd

Contrary to Percy's starting point, Lerato started his proof by clearly stating that his odd numbers would be called x and y. He then defined $x = 2a + 1$ and $y = 2a + 1$ and specified that the numerical context of a is that of an integer. This is a correct and well stated definition of an odd number. The challenge with Lerato's solution is that he defined x and y the same way, namely, both equal to $2a + 1$. While this is an acceptable definition, the implication is that Lerato is arguing on one case, where the two odd numbers are the same. Thus, by implication, he is now proving the narrow version of the required results. His proof will be fit to conclude that the product of two same odd numbers is always odd. Like Percy, Lerato's conclusion did not draw from the definition of an odd number. One would expect the last step of the proof to have the property of an odd number, which is two multiplied by an integer and adding the numeral 1.

Figure 6.

Lerato's Response to Task 2

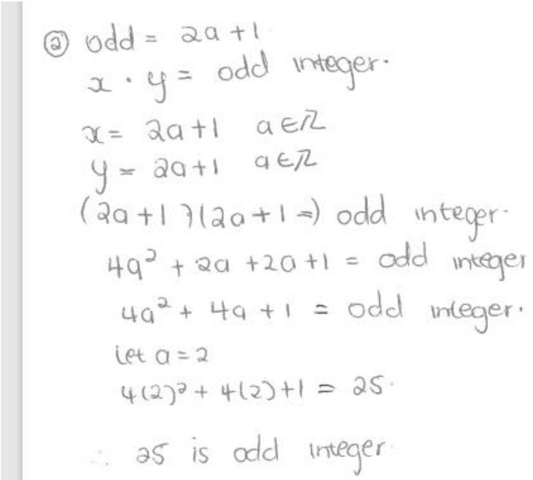


2. Given that the product of any two odd number = odd numbers
*Let one odd number = x
*Let the second one be = y
$\therefore x = 2a+1$ ~~$a \in Z$~~ $a \in R$
$y = 2a+1$ ~~$y \in R$~~ $a \in R$
$x \cdot y$ = odd number
$= (2a+1) \times (2a+1)$
$= 4a^2 + 2a + 2a + 1$
$= 4a^2 + 4a + 1$
~~$= 4a(a+2)+1$~~ (which is a odd number)
$= 4a^2 + 4a + 1$
$= 4a(a+1) + 1$ which is an odd number

Similar to Percy, Dimpho defined her two odd numbers to be the same, $x = 2a + 1$ and $y = 2a + 1$. Dimpho also made it clear that the numerical context is that of odd numbers. On step 3 of her proof, Dimpho wrote: $(2a + 1)(\ 2a + 1\ ) \rightarrow$ odd number. She concluded that the product is odd without any convincing arguments. This is clearly a writing problem as the student did not stop there but continued. The fact that she continued shows that the student is aware that she is not done with the proof, but her previous step suggests to the reader that the student claims the proof is complete or there is nothing to prove. Similar to both Lerato and Percy, Dimpho's conclusion did not in any way draw from the definition of odd numbers. Instead, Dimpho confused the whole proof by specifying that the randomly picked integers that she used at the beginning of her definition of odd numbers is now the numeral 2. That is, $x = 2(2) + 1$ = 5 and $y = 2(2) + 1 = 5$. Therefore, the whole effort that Dimpho put in her proof was reduced to just illustrating to the reader that (5)(5)=25 is an odd number.

Figure 7.

Dimpho's Response to Task 2



(2) odd = $2a+1$
$x \cdot y$ = odd integer.
$x = 2a+1 \quad a \in \mathbb{Z}$
$y = 2a+1 \quad a \in \mathbb{Z}$
$(2a+1)(2a+1 \Rightarrow)$ odd integer.
$4a^2 + 2a + 2a + 1$ = odd integer
$4a^2 + 4a + 1$ = odd integer.
let $a = 2$
$4(2)^2 + 4(2) + 1 = 25$.
$\therefore$ 25 is odd integer

The liminal rite of passage leads to feelings of inadequacy as well as reactions of euphoria. These emotional responses are especially powerful for students who did well in high school (Di Martino & Gregorio, 2019). In their interviews, both Dimpho and Lerato stated that they performed very well in their exit exams at secondary schools but now worried whether they could perform at the same level or above during their first year at university.

> *I have always done well in mathematics; I am good with calculations and solving proper maths problem, but this year was very difficult although I passed. I do not like writing explanations.* (Dimpho*)*
>
> *Many things had changed. I now doubt my mathematics ability and prefer working alone. I got distinction in grade 12, now I don't know what I will tell my parents and teachers should I fail. The first test knocked me off, mainly because of definitions—they were hard and carried lot of marks (25%) and I got literally zero on definitions. It is different for me to use words in mathematics. I enjoy calculations.* (Lerato)

It is unavoidable for students not to encounter this cultural shift as they enter university and dealing with the crisis is a necessary step for a successful passage instead of avoiding it (van Gennep, 1966. It is at this critical stage that university lecturers need to support students to overcome the crisis. The crisis needs to be managed in order for students to have better first year mathematics learning experiences (Klymchuk &Thomas, 2012). Lack of support for students during the liminal stage may lead to an unsuccessful rite of passage and result in students never reaching the incorporation stage and ultimately dropping out of mathematics courses during their first year at university. Even though students can drop out of university at any time during their career, most dropouts in mathematics courses happen during the first year (McGhie, 2012; Moodley & Singh, 2015).

Inconsistency in the use of Mathematical Notations

The third task was given three months into the course (incorporation stage), towards the end of the first semester as the teaching started in the last week of February. Students were now used to the university ways of writing mathematics and could tackle more advanced proofs which use highly specialised language and advanced methods of proofs that need double implications. The students were asked to prove Augustus De Morgan's law of sets which states that "the complement of the union of two sets is equal to the intersection of their complements". In the university mathematics discourse, different textbooks prescribed for first year mathematics courses use different notations to denote the same mathematical concepts (Morgan et al., 2014; Moodley & Singh, 2015). Also, different lecturers may use different notations in their teaching to represent the same concepts. What is common between textbook writers and mathematics lecturers is that whatever notations they chose to use, they do so consistently throughout the problem-solving process or in the process of writing the textbook or study notes. It is a standard practice endorsed by the community of mathematicians that you do not use different notations in the same textbook or in the process of solving one problem (Martínez & Dominguez, 2018).

In this study, the students switched notations in the process of solving the same problem. On the positive side, students started using inclusives such as "we", conditional imperatives "if" and started to speak with authority, "we must" (see Figures 8 and 9). The use of the imperative is one of the key characteristics of university mathematics communication and a sign that one is gaining membership to the community of university mathematicians by speaking the language of the community (Sfard, 2007; Wenger, 1998). The use of the first-person plural "we" allows students to move away from an absolutist image of mathematics as a system independent of human action (Engelbrecht & Harding, 2008; Gueudet, 2008). By using "we", the student brings the human element into the process of solving the problem and becomes personally involved in the process.

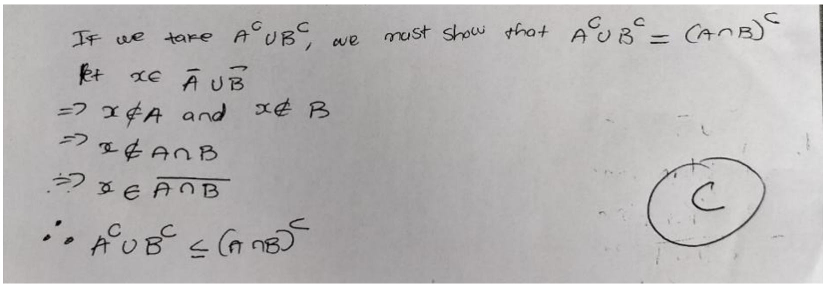
For task 3, the biggest challenge was the inability to use the same mathematical symbols consistently in the process of proof. Percy started his proof by taking the reader through the process and used words to explain the process using exact notations used by the lecturer in posing the problem. He gave an invitation to the reader "If we", then wrote "we must" show that $(A \cup B)^c$= $A^c \cap B^c$. But, surprisingly, the student was no longer showing what he promised the reader that he would show. He then took an element x from an unknown set called $\bar{A} \cup \bar{B}$, a completely different notation to the one used by the lecturer in posing the question (see Appendix A). While the new set was used to make an argument, the original set and notations were brought only at the conclusion: During the interview, Percy said:

The writing of different notations sometimes comes natural because they meant the same things.

The student did not see anything wrong with switching notations that mean the same concepts. Again, he did not see anything wrong with replacing the union with intersection and vice versa. Unlike school mathematics, university mathematics has many symbols with the same meaning (Gueudet, 2008). While both notations are acceptable, consistency is important. Mature writers use one notation consistently and do not confuse the reader. Inconsistency in the use of symbols suggests lack of mathematical writing maturity or mathematical deficiency (Martin, 2015). In many cases, as seen when grading mathematics at university, once the logic or meaning is lost in the previous step, the steps that follow will not make any sense (Durand-Guerrier et al., 2012; Engelbrecht & Harding, 2008).

Figure 8.

Percy's Response to Task 3


If we take $A^c \cup B^c$, we must show that $A^c \cup B^c = (A\cap B)^c$
let $x \in \bar{A} \cup \bar{B}$
$\Rightarrow x \notin A$ and $x \notin B$
$\Rightarrow x \notin A \cap B$
$\Rightarrow x \in \overline{A \cap B}$
$\therefore A^c \cup B^c \leq (A \cap B)^c$
C

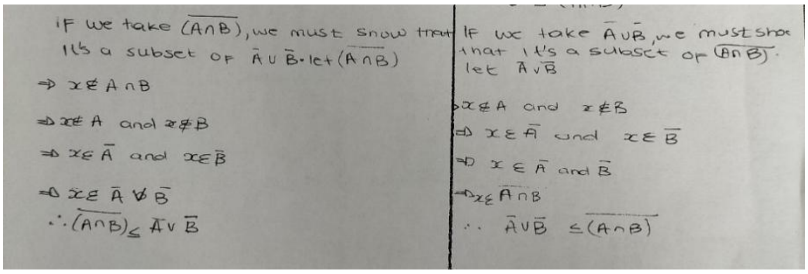
Although Lerato also changed the notations given by the teacher, the student did so consistently in his proof. Most importantly, the student presented clear narrative of what he intended to prove.

During the interview, Lerato said:

I was not even aware that I changed the notations given on the question paper; all I know is that my answer was correct. All I knew the two notations both mean complement.

Figure 9.

Lerato's Response to Task 3


If we take $(\overline{A\cap B})$, we must show that it's a subset of $\bar{A} \cup \bar{B}$. let $(\overline{A\cap B})$
$\Rightarrow x \notin A \cap B$
$\Rightarrow x \notin A$ and $x \notin B$
$\Rightarrow x \in \bar{A}$ and $x \in \bar{B}$
$\Rightarrow x \in \bar{A} \vee \bar{B}$
$\therefore (\overline{A\cap B}) \leq \bar{A} \vee \bar{B}$

If we take $\bar{A} \cup \bar{B}$, we must show that it's a subset of $(\overline{A\cap B})$. let $\bar{A} \vee \bar{B}$
$\Rightarrow x \notin A$ and $x \notin B$
$\Rightarrow x \in \bar{A}$ and $x \in \bar{B}$
$\Rightarrow x \in \bar{A}$ and $\bar{B}$
$\Rightarrow x \in \overline{A \cap B}$
$\therefore \bar{A} \cup \bar{B} \leq (\overline{A \cap B})$

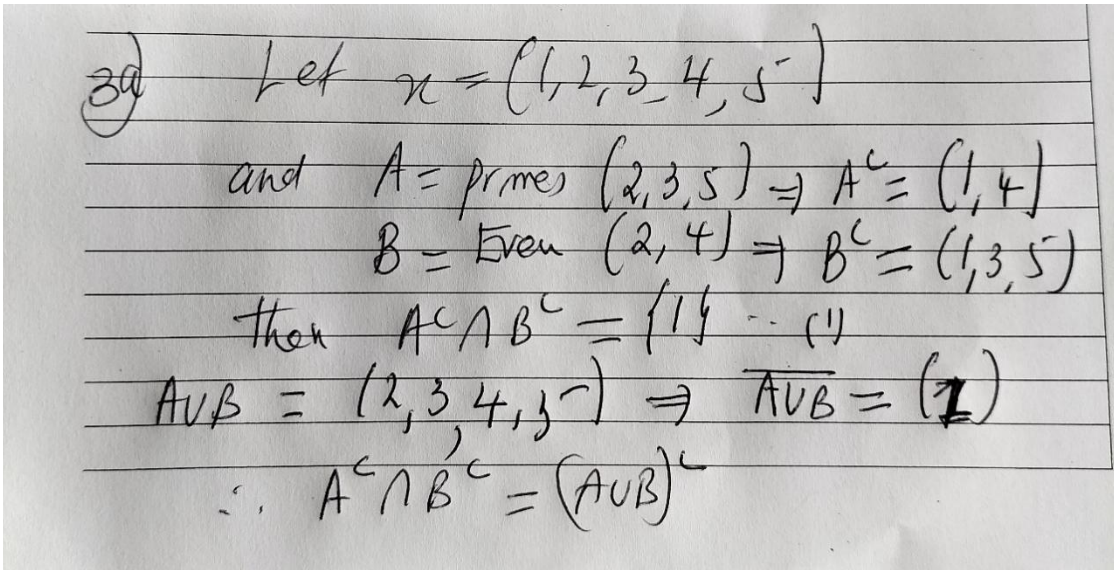
While both Lerato and Percy progressed well with being inducted to university mathematical writing rituals, in task 3, Dimpho got back to confusing illustration as proof, the same problem she evidenced during her first week at university. She was still in the school ways of showing mathematical facts. It is understandable that when students are faced with new situations, they recall easily from the already established ways of doing things (Engelbrecht, 2010; Engelbrecht & Harding, 2008). The student has spent many years at school and when encounting challenges the school culture of doing things kicks in (Engelbrecht & Harding, 2008; Sfard, 2007; Van Gennep, 1966). The influential organisation NCTM (1989) stresses the importance of communication in mathematics. According to the NCTM (1989), teaching mathematics should include learning to communicate and reason mathematically in writing form and taking care of numerical context.

Dimpho used specific sets to prove the general case. She defined her Universal set $X = \{1,2,3,4,5\}$ and A, B subsets of X with $A = \{2,3,5)$ and $B = \{2,4\}$. Then, she used A and B to show the required results, namely:

$$(A \cup B)^c = A^c \cap B^c.$$

Figure 10.

Dimpho's Response to Task 2



However, one specific case cannot be sufficient to claim general sets. Even if the student were to give a million examples, it would still not be sufficient to claim that the statement is always true. In the discourse of university mathematics, specific case/s can only be used to disprove, not to prove. To prove general results, students need to choose arbitrary elements and show that results are true for that randomly picked element. Then generality can be concluded. Students confused (1) as numbering the equation number and again the same notation was used to mean a set consisting of only one element (singleton). It confuses the reader as the same notation is used to mean different things. Like Percy and Lerato, in the second-last step, Dimpho switched from the notation $(A \cup B)^c$ to using $\overline{(A \cup B)}$ to mean the same thing.

CONCLUSION AND RECOMMENDATION

Traditionally, the challenges in mastering mathematics were attributed primarily to the cognitive demands inherent in mathematical content (Chandrasegaran, 2013; Engelbrecht, 2010). However, this study reveals a critical link between communication and writing skills and successful mathematics learning, particularly as students make the crucial transition from school to university-level mathematics. Proficiency in expressive mathematical language emerges as a critical factor for success in first-year university mathematics. The study identifies three distinct transitional challenges students face in mathematical writing: confusing illustrations for proofs, struggling to specify numerical contexts, and exhibiting inconsistent notational usage when addressing problems. During the separation stage, students often misconstrue illustration as proof, as university mathematics demands a rigorously formal approach with terms derived from formal definitions (Gueudet, 2008). The limited exposure to formal proof methods at the school level contributes to this challenge (Engelbrecht & Harding, 2008).

In the liminal stage, the study highlights a key challenge—students' difficulty in defining numerical contexts and drawing conclusions from established definitions. The incorporation stage witnesses students adopting first-person plural and authoritative narratives to signify their membership in the university mathematics community. However, they grapple with maintaining consistent notation in problem-solving, a crucial aspect impacting argument flow, logic, and communication with examiners, ultimately influencing their grades. The study underscores that the primary challenges in first-year university mathematics stem not from the abstract nature of the content but from inadequate mathematical writing, where students struggle to articulate their thought processes in written form. To address these issues, teachers must be cognizant of the challenges associated with mathematical writing during the school-to-university transition, as ignorance of these issues can have severe consequences for students.

In light of these findings, a paradigm shift is advocated, challenging traditional mathematical pedagogies and placing writing at the core of first-year university mathematics education. The study recommends explicit instruction in mathematical writing to guide students through this academic rite of passage. First-year mathematics lecturers are encouraged to integrate writing activities into their instruction, fostering both mathematical thinking and comprehension. To ease the cultural shock experienced by students during this transition, a shift in focus is suggested—from an emphasis on content coverage to aiding students in constructing their new identities, teaching them the customs, traditions, and proper methods of constructing and writing mathematical proofs at the university level. Furthermore, the study proposes avenues for future research, suggesting the replication of this study or exploration of mathematical writing challenges as students progress from their first to the second year of university mathematics. This ongoing inquiry will contribute to a comprehensive understanding of the evolving challenges in mathematical writing throughout students' academic journeys and navigating the cultural transitions in mathematical discourses.

Limitations

The nature of my research was qualitative and the results from qualitative study are not generalisable, as the context of the study pays a huge role in the results of the analysis. The study was a small-scale study from one first-year module at one university in one country. In this study, the main limitation was the use of one set of students, from one teacher, one module, at one university. Therefore, the results of this study cannot be generalisable for all modules or other universities as they are context specific. . Also, the study did not include the interviews with the students after their final examination. The time of the students' exams and their unavailability to take part in the interviews afterward led to the elimination of post-final examination interviews.

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APPENDIX A

TASK 1

Prove that the sum of any two odd integers is always an even integer.

TASK 2

Prove that the product of any two odd integers is always an odd integer.

TASK 3

Let A and B be any arbitrary non-empty sets. Prove De Morgan's law of sets which states that the complement of the union of two sets is always equal to the intersection of the complements of the two sets.

$$(A \cup B)^c = A^c \cap B^c$$